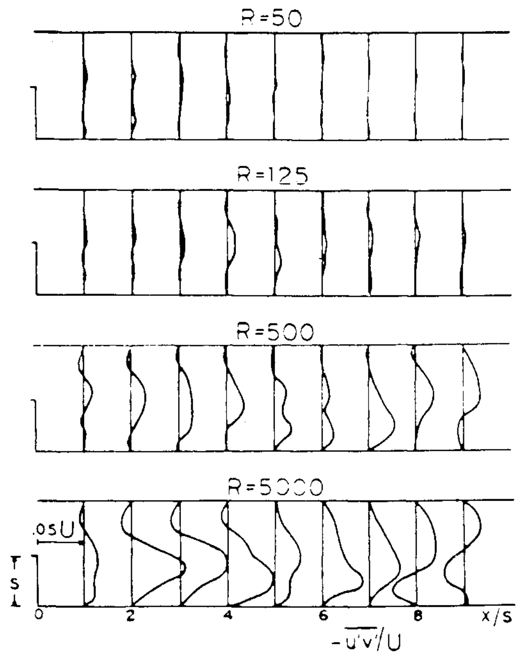

Fig. 8 Turbulent shear stress $-\overline{u'v'}$ for the four values of Reynolds number considered in Figs. 5–7.

velocity reverses its sign close to the wall, increases with Reynolds number to a maximum value of about 9S at $R=500$, then decreases to 6S at $R=5000$; and 2) the maximum value of the recirculation velocity increases at higher Reynolds number, and the area containing fluid that moves with negative velocity becomes wider. Figures 6–8 show the average values of the second moments of velocity and their cross-correlation. Results indicate that 1) the values of $\overline{v'v'}$ and $-\overline{u'v'}$ increase with R, locally reaching a maximum around the boundary of the recirculation zone, i.e., at the shear layer, where the momentum flux across the layer is transported by the passing eddies; 2) at transition and beyond, $R>500$, the gradient of the turbulent shear stress reaches zero on the wall at the reattachment point section; 3) the maximum computed value of $\sqrt{\overline{u'u'}}=0.25$ occurs at $R=5000$, whereas Dust and Tropea[10] show a wide scatter of experimental data at that range, with a mean value of about 0.2, and Atkins et al.[13] present experimental results that exhibit the same double-peaked profiles of Fig. 6 close to the step; and 4) the maximum values of $-\overline{u'v'}$ reach 0.05 and stay almost unchanged until the reattachment point, whereas experimental results[13] show a maximum of about 0.02.

Conclusions

The structure of the recirculation zone forming behind a rearward-facing step in a channel is computed for Reynolds number in the 50–5000 range, using the random vortex method. Results are analyzed in terms of instantaneous streamlines, average velocity profiles, and turbulence statistics. Four distinct flow regimes are identified. At very low Reynolds numbers, the flow is viscous, and the recirculation zone is formed of one stationary eddy at the corner of the step. As the Reynolds number increases, an eddy may detach and decay as it moves downstream. At moderate values, the flow reaches a state of transition, with eddy shedding at the step. At high Reynolds numbers, the flow exhibits turbulent behavior with a continuous process of eddy formation and pairing. The recirculation zone length increases with Reynolds number, reaching a maximum at transition and then decay to a shorter length at the turbulent range. Turbulence statistics show a rise at transition.

Acknowledgments

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Experimental Evaluation of Approximations for $\overline{w^2}$ and $\overline{vw^2}$

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Introduction

ASSUMPTIONS involved in the assessment of any physical situation should be reduced to a minimum in order to increase knowledge of a subject. The purpose of this Note is to evaluate assumptions that are usually made for some moments of turbulence fluctuating velocity. One of these moments requires additional time to measure, whereas the other is difficult to measure and is time consuming. The

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following assumptions in turbulence regarding $\overline{w^2}$ and $\overline{vw^2}$ are generally made by researchers[1-4]:

$$\overline{w^2} = 0.5(\overline{u^2} + \overline{v^2}) \tag{1}$$

$$\overline{vw^2} = 0.5(\overline{u^2 v} + \overline{v^3}) \tag{2}$$

where u, v, and w are the longitudinal, normal, and transverse turbulent fluctuating velocities, respectively. The overbar indicates the mean value of the quantity. Equations (1) and (2) affect the mean and turbulent transport of the turbulent kinetic energy. They are examined in light of experimental measurements in three flows, namely, boundary layer, fully developed pipe flow, and diffuser flow. Details regarding these three flows are given in Refs. 5 and 6.

The measurement of triple correlation, $\overline{vw^2}$, first outlined by Townsend[7] and later followed by others,[8,9] was adopted in the present work. Briefly, the method requires that the X-wire be placed in the flow in such a way that its plane is parallel to the flow and it is at an angle of 45 deg to both the y and z axes. Detailed derivation is given in Ref. 8 (there is a misprint in the final form in this reference—$\overline{v^2}$ was incorrectly replaced by $\overline{u^2}$).

Results and Discussion

Figure 1 shows the values of $\overline{w^2}/(\overline{u^2} + \overline{v^2})$ for boundary layer, fully developed pipe flow, and diffuser flow. The results for boundary layer are for $R_\theta = (U_\infty \theta)/\nu = 2.76 \times 10^3$, in which U_∞ is its freestream velocity, θ its momentum thickness, and ν the kinematic viscosity of a fluid. In this figure, a value of $\overline{w^2}/(\overline{u^2} + \overline{v^2})$ equal to 0.4 is displayed instead of the value of 0.5 that is usually assumed, except for some variations in the intermittent region ($y/\delta < 0.4$, where y is the distance from the wall and δ is the boundary layer thickness). The results for fully developed pipe flow are given for two values of the pipe Reynolds number, $Re = 1.14 \times 10^5$ and 2.31×10^5; and for two pipe lengths L in term of diameter D, given by $L/D = 78$ and 98. The Reynolds number is based on mean velocity, U_b, at a cross section, the pipe diameter D, and kinematic viscosity, ν, i.e., $Re = (U_b D)/\nu$. The value of $\overline{w^2}/(\overline{u^2} + \overline{v^2})$ is not equal to 0.5, but has a value of 0.4 near the pipe wall, 0.35 at $y/R = 0.15$ (R is the radius of the pipe), and 0.41 at the center of the pipe (see Fig. 1). The two quantities reported in this Note have been measured at six different axial locations in a diffuser, but, for conciseness, only one location that is thought to be representative of the behavior of these two quantities has been selected for presentation. This position is located at a distance of 42 cm from the inlet of the diffuser. The result for $\overline{w^2}/(\overline{u^2} + \overline{v^2})$ shown in Fig. 1 decreases rather rapidly from a value of 0.48 near the wall of the diffuser to 0.33 at $y/R_{\rm LOC} = 0.8$ ($R_{\rm LOC}$ is local radius) and monotonically increases to 0.38 at the axis of the diffuser.

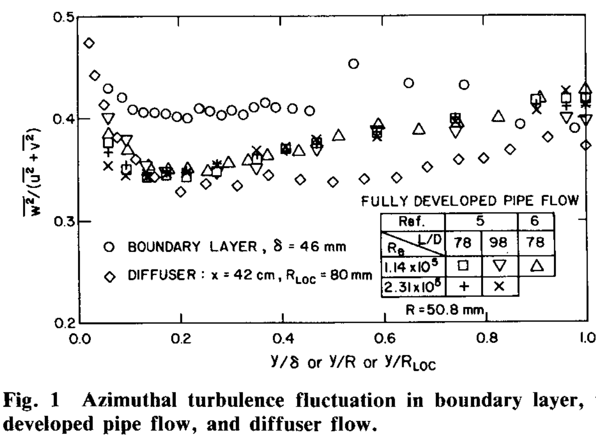

Fig. 1 Azimuthal turbulence fluctuation in boundary layer, fully developed pipe flow, and diffuser flow.

Figure 2 shows the results of third moment, $\overline{vw^2}/(\overline{u^2 v} + \overline{v^3})$, for boundary layer and fully developed pipe-flow. In case of boundary layer, its value starts to drop linearly from more than 1 near the wall to 0.2 at $y/\delta = 0.4$ and remains at this value throughout the boundary layer. In the same figure, the pipe flow results show a value approaching 1 near the wall, then decreasing to 0.2 at $y/R = 0.4$ where it remains until reaching the axis of the pipe.

Both of the quantities, i.e., $\overline{w^2}/(\overline{u^2} + \overline{v^2})$ and $\overline{vw^2}/(\overline{u^2 v} + \overline{v^3})$, shown in Figs. 1 and 2 clearly demonstrate that the flow in the pipe is fully developed and is independent of Reynolds number over the range tested. In addition, credence is added to the measurements since two of the present authors[5,6] have obtained these results independently over a three year period.

Since the sign of $\overline{vw^2}$ is not the same as $(\overline{u^2 v} + \overline{v^3})$ across the radius in the diffuser flow (as they are in fully developed pipe flow and boundary layer), the results for $\overline{vw^2}$ and 0.5 $(\overline{u^2 v} + \overline{v^3})$ are plotted with different symbols in Fig. 3. It can be seen from the figure that these quantities are quite far apart from each other, from $y/R_{\rm LOC} = 0.2$ to the axis of the diffuser. A similar trend has been observed at other locations on the diffuser.

The expression given by Eqs. (1) and (2) are not valid, as demonstrated in Figs. 1–3 for the present measurements in three turbulent shear flows. The above inference is also corroborated by others.[10-12] In particular, Wood and Bradshaw[10] obtained $\overline{vw^2} = 0.20(\overline{u^2 v} + \overline{v^3})$ across the whole turbulent mixing layer. It can be inferred (from Table 5.2 by Townsend[11]) that $\overline{w^2}/(\overline{u^2} + \overline{v^2}) = 0.37$ and 0.63 for the laboratory equilibrium layer and the atmospheric equilibrium layer, respectively. Finally, Andreopoulos and Wood (Fig. 9)[12] compared $\overline{w^2}$ with $0.5(\overline{u^2} + \overline{v^2})$ and found that the approximation $\overline{w^2} = 0.5(\overline{u^2} + \overline{v^2})$ was erroneous.

From the measurements of Laufer,[14] Lawn[13] deduced that $\overline{vw^2} \cong 0.5\overline{v^3} \cong 0.5\overline{u^2 v}$ and, in turn, used $\overline{vw^2} = 0.25(\overline{u^2 v} + \overline{v^3})$; Nakagawa et al.[15] approximated $\overline{vw^2} = \overline{v^3}$.

The preceding discussions of the assumptions concerning two parameters, $\overline{w^2}$ and $\overline{vw^2}$, clearly indicate that none of

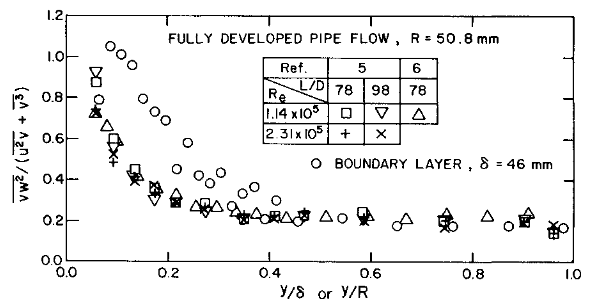

Fig. 2 Third moment $\overline{vw^2}$ of fluctuating velocity in boundary layer and fully developed pipe flow.

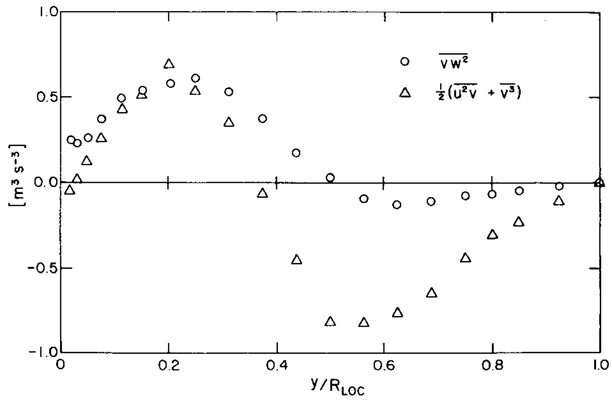

Fig. 3 Third moment $\overline{vw^2}$ of fluctuating velocity in diffuser flow.

the assumptions really work in all the turbulent flows. Hence, generalization in turbulence is very risky. In other words, the quantity required has to be measured.

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Conical, Separated Flows with Shock and Shed Vorticity

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Introduction

THIS investigation deals with the highly vortical flow about circular cones at supersonic speeds and high angle of attack. The author has shown that the shock system of these flows can cause separation and a spiral vortex near the lee plane of the cone.[1] Earlier, Salas[2] showed two-dimensional Euler solutions where shock vorticity had also induced separation. The numerical investigation of Ref. 1 showed that the crossflow shock can produce large entropy gradients and vorticity sufficient to cause separation and a spiral vortex. In addition to shock vorticity, a separating boundary layer can shed vorticity into an otherwise irrotational flow. This phenomenon has been studied with inviscid models for many years. Smith[3] developed a model for shedding vorticity from the surface of a smooth body into the flowfield. The present author[4] was the first to use this model in conjunction with the Euler equations to shed vorticity from both primary and secondary crossflow separation points. An extension of the vorticity shedding model of Ref. 3 to the situation of supersonic crossflow is used in the present investigation. In the case of shock-induced separation, the solution to the Euler equation is unique in that the separation point location is computed along with the shock system and flowfield. On the other hand, the separation point must be prescribed in the case of shed vorticity. There exists a range of separation point locations corresponding to varying amounts of vorticity being shed. This Note presents the results of an investigation of the relationship between shock-vorticity-produced separation and shed-vorticity-produced separation.

Discussion

This study was conducted by numerically evaluating Euler solutions for the flow about a 5 deg cone varying the specified separation point location. The freestream conditions were fixed at $M_\infty = 4.25$ and $\alpha = 12.35$ deg. The flow is conical so that only the crossflow plane (i.e., any plane normal to the cone axis) need be considered. This plane intersects the separation line at a point which will be referred to as the separation point. At the large angle of attack considered here the flow in the crossflow plane is supercritical and so a crossflow shock is present. With no vorticity shed from the cone surface, this crossflow shock is strong enough to produce enough vorticity to cause separation. In addition, shedding vorticity from the cone surface does not eliminate the crossflow shock so that both sources of vorticity are present. Figures 1 and 2 show the crossflow streamlines and isobars, respectively, for the flow with no vorticity shed from the body. Separation for this case is computed to be at $\theta_s = 151.3$ deg from the wind plane. Figure 1 indicates that the separating streamlines leave the surface at a large angle (57 deg in this particular case) relative to the surface. When only shock vorticity is present there is no jump in crossflow velocity at the separation point, consistent with the fact that no vorticity is being shed from the surface. In the case of the shock vorticity alone, the crossflow stagnates on both sides of the separating streamline, and this streamline leaves the surface at a large angle relative to it. Smith[3] proved that in order to shed vorticity into an otherwise irrotational flow there must be a jump in crossflow velocity at the separation point; the crossflow stagnates only on the lee side of the separating steamline. It should be pointed out that in the computational results presented here, all crossflow shocks are captured. Figure 2 indicates that the shock is captured very sharply (see the closely spaced isobars). Additionally, these captured shock results compare well with the shock fit results of Ref. 1.

The model used to shed vorticity from the cone surface follows the work of Smith.[3] The model is implemented in the computational scheme by using a double grid point at the separation point. One grid point is assumed on the flow side of the vortex sheet and the other is assumed on the body side. In the case of subsonic crossflow the conditions on the flow side of the sheet are computed as any other body point, satisfying only the flow tangency condition. The crossflow is assumed to stagnate on the body side of the sheet; with the pressure given from the flow side of the sheet all conditions at this point can be computed. These conditions result in the sheet leaving the

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